INTEGRATED SENSING AND COMMUNICATION BEYOND I.I.D. AND STATIC MODELS—PART II

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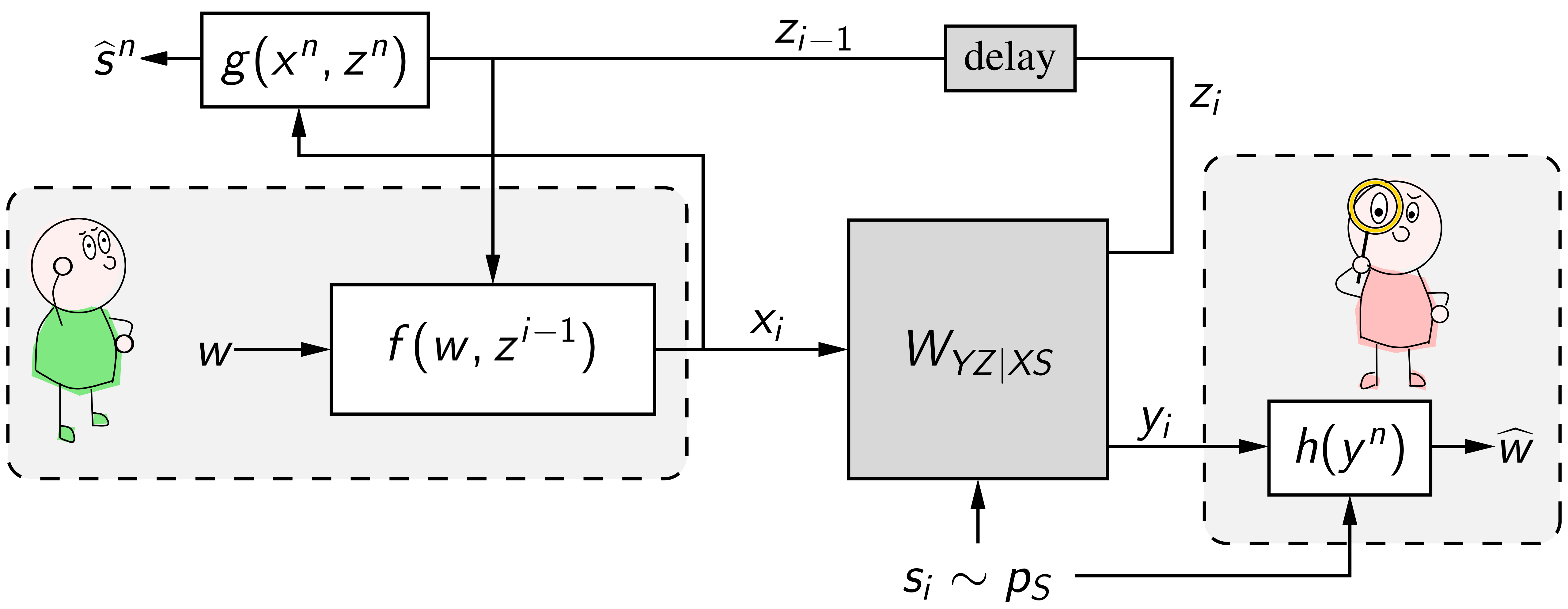


- **Integrated Sensing and Communication MODEL**
  - **Metrics:**

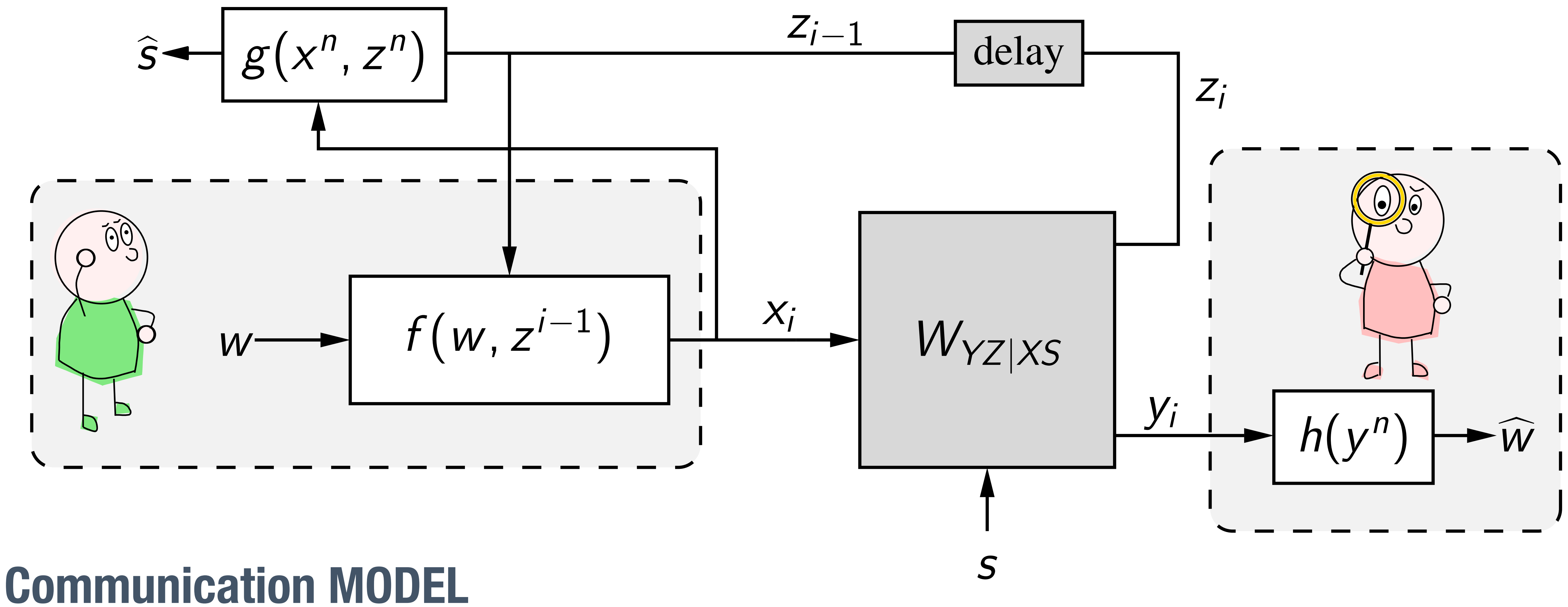
$$P_e^{(n)} = \mathbb{P}\left(W \neq \widehat{W}\right) \quad d^{(n)} = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[d(S_i, \widehat{S}_i)\right]$$

  - **Reduces again to a rate-distortion tradeoff problem**
    - No prediction capability (state is i.i.d.)
    - Tradeoff governed by type of codewords $C(D) = \max_{p_X \in \mathcal{P}_D} I(X; Y|S)$

$$\mathcal{P}_D \triangleq \{\, p_X : \sum_x p_X(x) d^*(x) \leqslant D \,\} \quad d^*(x) \triangleq \mathbb{E}\left[d(S, \widehat{s}^*(X, Z)|X = x)\right] \quad \widehat{s}^*(x, z) \triangleq \operatorname{argmin}_{s'} \sum_s p_{S|XZ} d(s, s')$$

An Information-Theoretic Approach to Joint Sensing and Communication, Ahmadipour, Kobayashi, Wigger, Caire, *IEEE Transactions on Information Theory*, May 2022

An Information-Theoretic Approach to Collaborative Integrated Sensing and Communication for Two-Transmitter Systems, Mehrasa Ahmadipour and Michèle Wigger, *IEEE Journal on Selected Areas in Information Theory*, 2023



- **Integrated Sensing and Communication MODEL**
  - State-dependent Discrete Memoryless Channel (Compound Channel) with state $s \in \mathcal{S} \quad |\mathcal{S}| < \infty$
  - Encoder: $f_i : [1;M] \times \mathcal{Z}^{i-1} \to \mathcal{X} \quad \forall i \in [1;n]$, decoder: $h : \mathcal{Y}^n \to [1;M]$, estimator: $g : \mathcal{X}^n \times \mathcal{Z}^n \to \mathcal{S}$
- **Performance Metrics**
  - Communication and detection error probability:
  $$P_{\mathrm{c}}^{(n)} \triangleq \max_{s\in\mathcal{S}} \max_{w\in[1;M]} \mathbb{P}(h(Y^n) \neq w | W = w, S = s) \quad P_{\mathrm{d}}^{(n)} \triangleq \max_{s\in\mathcal{S}} \frac{1}{M} \sum_{w=1}^{M} \mathbb{P}(g(Z^n) \neq s | S = s, W = w)$$
  - Rate: $R \triangleq \frac{1}{n} \log M$ and detection error exponent $E_{\mathrm{d}}^{(n)} \triangleq -\frac{1}{n} \log P_{\mathrm{d}}^{(n)}$

📚 **Rate and Detection Error-Exponent Tradeoffs of Joint Communication and Sensing,** Meng-Che Chang et al., *Proc. of IEEE Int. Symp. on JC&S*, 2022
📚 **Joint Communication and Binary State Detection,** Joudeh and Willems, *IEEE Journal on Selected Areas in Information Theory*, 2022
📚 **On Joint Communication and Channel Discrimination**, Wu and Joudeh, *Proc. of IEEE International Symposium on Information Theory*, 2022
📚 **Rate and Detection-Error Exponent Tradeoff for Joint Communication and Sensing of Fixed Channel States,** Meng-Che Chang et al., *IEEE Journal on Selected Areas in Information Theory*, 2023
📚 **On the Fundamental Tradeoff of Integrated Sensing and Communications Under Gaussian Channels**, Yifeng Xiong et al., *IEEE Trans. on Info. Theory*, 2023

- **Relative Coherences in ISAC Models**
  - Time range defined for $\forall i \in [1,n]$
  - Symbol coherence τ_c, sensing coherence τ_b and state coherence τ_s follow the relation: $\tau_c << \tau_b << \tau_s$.

	i.i.d States[1]	Fixed State[2]	Fixed State with different sensing/comm coherence (Today's Topic [4][5])	Block-Fading (Current ongoing research) *[3]
Symbol (Comm) Coherence	l	l	l	l
Sensing (Beam) Coherence	l	l	τ_b	τ_b
State Coherence	l	n	n	τ_b
Code Coherence	n	n	n	n

- **[1] An Information-Theoretic Approach to Joint Sensing and Communication,** Ahmadipour, Kobayashi, Wigger, Caire, *IEEE Transactions on Information Theory*, May 2022
- **[2] Rate and Detection-Error Exponent Tradeoff for Joint Communication and Sensing of Fixed Channel States,** Meng-Che Chang et al., *IEEE Journal on Selected Areas in Information Theory*, 2023
- **[3] On the Fundamental Tradeoff of Integrated Sensing and Communications Under Gaussian Channels**, Yifeng Xiong et al., *IEEE Trans. on Info. Theory*, 2023
- **[4] Rate-Reliability Region of Sequential Beam Alignment and Communication**, Guo and Bloch, *submitted to IEEE Information Theory Workshop, Apr. 2025*
- **[5] Sequential Joint communication and Sensing of Fixed States**, Meng-Che Chang et al., *IEEE Information Theory Workshop, 2023*

- **Noisy Beam-Pointing Channel Model**
  - B hypothesis representing unique angular domains.
  - For each $t' \in [1,n/\tau_b]$, $Y_{\tau_b(t'-1)+1:\tau_b t'} = S \cdot X_{\tau_b(t'-1)+1:\tau_b t'} \oplus N_{\tau_b(t'-1)+1:\tau_b t'}$, where $S \in \mathcal{S} = \mathbb{F}_2^{1\times B}$, $Z_{(t'+1)\tau_b} = \mathbf{1}\{\mathbf{1}^T Y^{\tau_b} > T\}$
  - Example Input State-Space vector $X^{\tau_b} = \begin{pmatrix} x_1(w) & x_2(w) & \dots & x_{\tau_b}(w) \\ x_1(w) & x_2(w) & \dots & x_{\tau_b}(w) \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{pmatrix}$, $a_{t'} = \{1,2\}$
- **Active Sequential ISAC Model**
  - Target resolution: $\delta = \Theta/B$, Message set $\mathcal{M} = \cup_{k=1}^{\infty} \mathbb{F}_2$
  - Number of message bits transmitted by t: $\phi_t : \mathcal{X}^{t-1} \times \mathcal{Z}^{t-1} \mapsto \mathbb{N} : (X^{t-1}, Z^{t-1}) \mapsto M_t$
  - Stopping rule $h_t : \mathcal{X}^t \times \mathcal{Z}^t \mapsto \Omega^B \times \mathcal{Q} : (X^t, Z^t) \mapsto \Omega_t^B \times Q_t$
  - Encoder $f_t : \mathbb{F}_2^{M_t} \times \mathcal{X}^{t-1} \times \mathcal{Z}^{t-1} \mapsto \mathcal{X} : (W^{M_t}, X^{t-1}, Z^{t-1}) \mapsto X_t$, Decoder $g : \mathcal{Y}^\tau \mapsto \mathcal{M} : Y^\tau \mapsto \hat{W}$
- **Performance Metrics**
  - Detection error $P_d^n \triangleq \max_{w\in\mathcal{M}} \mathbb{P}(\hat{S} \neq S \,|\, W = w)$, Communication error $P_c^n \triangleq \max_{w\in\mathcal{M}} \mathbb{P}(g(Y^\tau) \neq w[1;M_\tau] \,|\, W = w)$.
  - Rate and detection error exponent $R^n \triangleq \dfrac{M_\tau}{n}$ $\quad E_d^n \triangleq -\dfrac{1}{n} \log P_d^n$.

📚 **On the Capacity of Gaussian "Beam-Pointing" Channels with Block Memory and Feedback,** Siyao Li et al, *Proc. of 2024 ISIT*, (2024)

Definition: Achievability

A $(\delta, \{\phi_t\}_{t\geq 1}, \{h_t\}_{t\geq 1}, \{f_t\}_{t\geq 1}, g)$ policy is (R, E) achievable if for any $\epsilon_1, \epsilon_2, \epsilon_3 > 0$, there exists $n(\epsilon_1, \epsilon_2, \epsilon_3)$ such that

$$\max_{w\in\mathcal{M}} \mathbb{E}[\tau] \leq n,$$
$$\min_{w\in\mathcal{M}} \mathbb{P}(R^n \geq R) \geq 1 - \epsilon_1,$$
$$P_c^n \leq \epsilon_2,$$
$$E_d^n \geq E - \epsilon_3,$$

Theorem: Rate and Exponent Region

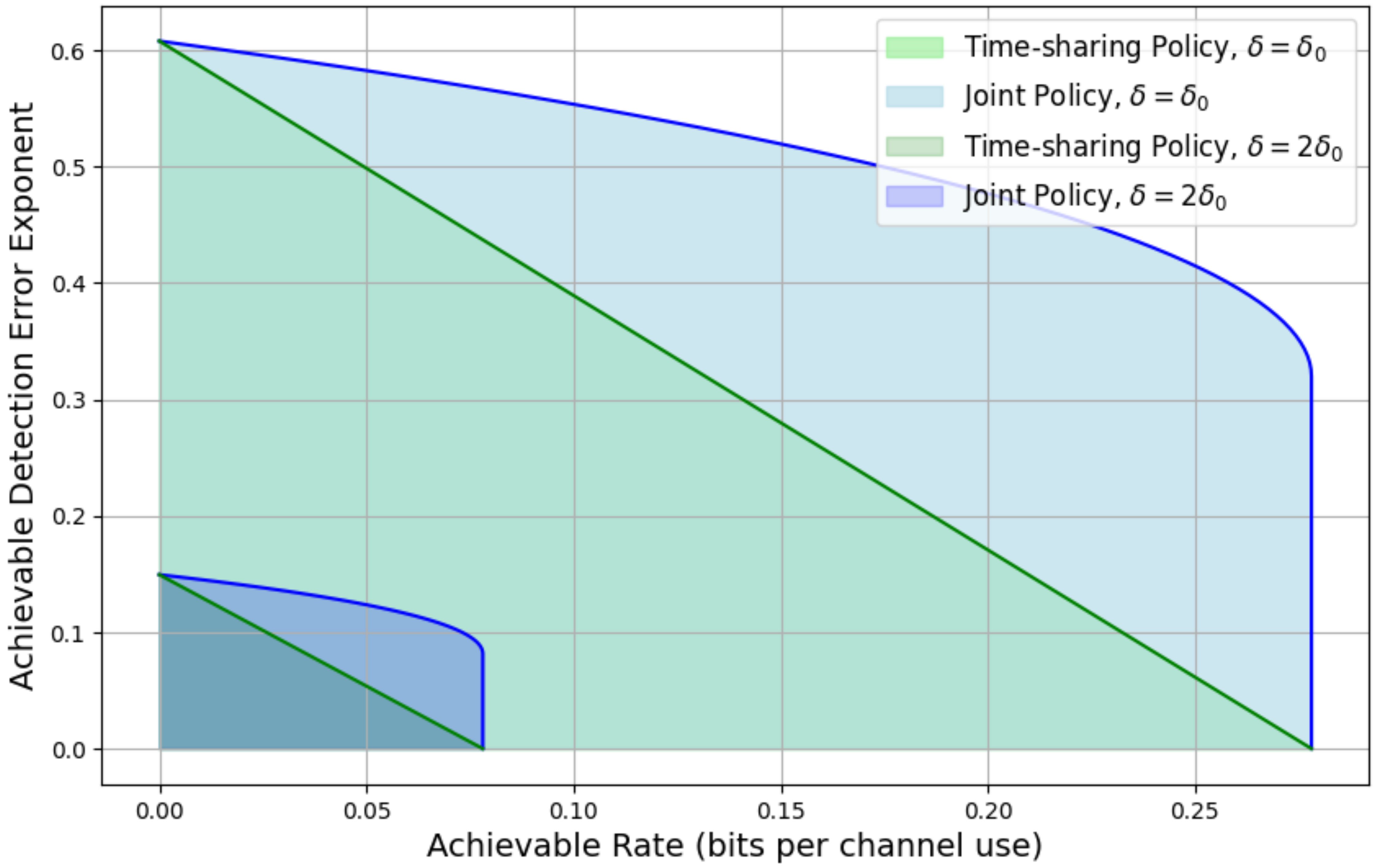
$$\mathscr{C} = \bigcup_{P_X\in\mathscr{P}_\mathcal{X}} \left\{ \begin{array}{l} (R,E) \in \mathbb{R}_+^2 : \\ R \leq \mathbb{I}\left(P_X, W_{Y|X}(p_1^{(\delta)})\right) \\ E \leq \dfrac{1}{\tau_b}\mathbb{D}\left(p_1^{(\delta)}(\alpha)\,||\,1 - p_1^{(\delta)}(\alpha)\right) \end{array} \right\},$$

Where $p_{|a_{t'}|}^{(\delta)}(\alpha) = \mathbb{P}\left(Z_{(t'+1)\tau_b} \neq \mathbf{1}\left\{S \cdot X_{t'}^{\tau_b} \neq 0^{1\times\tau_b}\right\}\right)$.

📚**Active Learning and CSI Acquisition for mmWave Initial Alignment,** S. Chiu et al., *IEEE Journal on Selected Areas in Communications*, (2019)

- **We use an non-asymptotic value of $\tau_b = 50$**
  - Assuming a phased array with beam switching rate 200Hz, baud rate 10Gbps.
  - Assume a base target resolution δ_0 such that $p_1^{(\delta_0)} = 0.2, p_1^{(2\delta_0)} \approx 0.337$, based on $p_1^{(\delta)} \sim Q(1/\delta)$.
  - By approximating the received weight of codewords using Gaussian distribution and treating two errors as result of threshold test equally we have,

$$p_1^{(\delta)}(\alpha) = 2Q\left(\frac{\alpha\tau_b(1-2p_1^{(\delta)})}{2\sqrt{\alpha\tau_b p_1^{(\delta)}(1-p_1^{(\delta)})}}\right)$$



Spatio-temporal filtering: Precise beam control using fast beam switching, Paidimarri and Sadhu, *IEEE Radio Frequency Integrated Circuits Symposium, Jun, 2020.*

- **Sketch of Proof:**
  - By Active Sequential Hypothesis Testing **[Naghshvar-Javidi'13]**, select ISAC action based on following

$$\eta^{(a)} = \begin{cases} \underset{\lambda\in\mathbb{P}(\mathcal{A}_B)}{\text{argmax}} \ \underset{\hat{\rho}\in\mathbb{P}(\Omega^B)}{\min} \sum_{a\in\mathcal{A}_B} \lambda_i^{(a)} \mathbb{D}(q_i^a \,||\, \sum_{j\neq i} \frac{\hat{\rho}_j}{1-\hat{\rho}_i} q_j^a) & \text{if } \exists i \ \rho_i \geq \rho^*, \\ \underset{\lambda\in\mathbb{P}(\mathcal{A}_B)}{\text{argmax}} \ \underset{i\in\Omega^B}{\min} \ \underset{\hat{\rho}\in\mathbb{P}(\Omega^B)}{\min} \sum_{a\in\mathcal{A}_B} \lambda_i^{(a)} \mathbb{D}(q_i^a \,||\, \sum_{j\neq i} \frac{\hat{\rho}_j}{1-\hat{\rho}_i} q_j^a) & \text{if } \max_{i\in\Omega^B} \rho_i < \rho^* . \end{cases}$$

  - Analysis of Stopping Time: Formulate stopping rules based on log-ratio posterior belief

$$U_i(t') = \log\left(\frac{\rho_i(t')}{1-\rho_i(t')}\right) - \log\left(\frac{\rho^*}{1-\rho^*}\right)$$ against exponent in Theorem to prove $\mathbb{E}[\tau] \leq n$

  - Analysis of Detection Exponent: By **[Naghshvar-Javidi'13]** Section 3.5
  - Analysis of Communication Error Probability: Using Union Bound to express the error using three events: 1. Decoding error in first phase. 2. Decoding error in second phase. 3. The wrong hypothesis crosses the threshold. Using standard constant composition code results and a martingale concentration lemma (next page) to bound each term vanishingly small.

📚**Active Sequential Hypothesis Testing,** Naghshvar and Javidi., *Annuls of Statistics* (2013)
📚**Extrinsic Jensen-ShannonDivergence: Applications to Variable-Length Coding,** Naghshvar et al.' *IEEE Transactions on Information Theory (2015)*

- **Sketch of Proof:**
  - Analysis of Rate:
    - We first formulate the expected number of message bits transmitted by time t using the expected number of bits transmitted within each block multiplied by the probability of that block in high/low confidence phase.
    - We then use martingale property to log-ratio posterior belief to prove the concentration on probability of transmission in first phase (bottom of the page).
    - Use Doob's decomposition trick (martingale difference) to to show that the stochastic process of number of message bits minus its expectation is a martingale with bounded increase.
    - Finally, Azuma's inequality bounds the number of message bits transmitted by time t close to its expected value.

Lemma: Martingale Concentration

Consider the sequence $U_{\tilde{i}}(t'), t' \in [0, \lfloor \tau/\tau_b \rfloor]$, there exists constants satisfying $0 < K_1 < K_2 < \infty$ such that

$$\mathbb{E}[U_{\tilde{i}}(t'+1) \,|\, \mathscr{F}_{t'}] \geq U_{\tilde{i}}(t') + K_1,$$
$$|\, U_{\tilde{i}}(t'+1) - U_{\tilde{i}}(t')\,| \leq K_2,$$

where $\mathscr{F}_{t'} = \sigma\{\boldsymbol{\rho}(t'), A(t'), Z(t'), t' \in [0, \lfloor \tau/\tau_b \rfloor]\}$.

Consider uniform prior and Markov stopping time, then $\mathbb{P}(U_{\tilde{i}}(n^{1/4}) < \theta) \leq e^{-n^{1/4}\iota}$ for $\iota, \theta > 0$.

- **Sketch of Proof:**
  - Assume that (R, E) is achievable
  - Applying change of variable by considering the relative timing of sensing/comm event $t' = t/\tau_b$, the converse of detection exponent follows by using **[Naghshvar-Javidi'13]** Corollary 3.5
  - By considering a set of BSCs indexed by cardinality of sensing action, the expected number of message bits transmitted by stopping time is upper bounded best compound channel result, which is when $|a| = 1$ **[Csiszar-Korner '11].**
  - Use Markov's inequality to convert expected number of transmitted message bits to a probabilistic constraint on rate.
  - Taking union of all type P_X.

- **CONCLUSION**
  - Timing of wireless events plays an important role in modeling
  - One can hope to adapt just as well on longer sensing coherence
  - In target detection use case, transmitter can hope to zoom in as quickly as possible to enable best exponent and rate as if direction is known in hindsight.

- **Future Works**
  - Regret-based analysis for block-fading ISAC model
  - Bringing sensing security into the picture

THANK YOU!