DETECTION PROBABILITY AND RATE TRADEOFF IN MIMO-OFDM ISAC SYSTEM WITH IMPERFECT CHANNEL INFORMATION

Sidong Guo[1], Xiangyu Li[2,3], Qingqing Wu[2], Bodong Shang[3]
[1]School Of Electrical & Computer Engineering, Georgia Institute Of Technology, Atlanta, USA
[2]Department Of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China
[3]Eastern Institute For Advanced Study, Eastern Institute Of Technology, Ningbo, China

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SPEAKER: XIANGYU LI

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OUTLINE

- **Background & Motivation**
- System Model
  - MIMO-OFDM ISAC Model
  - Performance Metrics
- Detection Probability Derivation
- Simulations & Numerical Results
- Conclusion

BACKGROUND AND MOTIVATION

- Integrated sensing and communication (ISAC) integrates the traditionally distinct domains of sensing and communication, enabling systems to jointly transmit data and gather valuable environmental information.

- **Features**
  - Wireless networks with enhanced capabilities: improved environment awareness, increased resilience to interference, efficient resource utilization
  - Recent researches have explored various facets: signal processing tools [Zhang et al. ’21a], network and system architecture [Zhang et al. ’21b], fundamental sensing and communication performance tradeoffs [Li et al. ’24]
  - Sensing for estimating channels, and detecting targets
  - Performance bounds between communication rate and probability of detection (PD) [Liu et al. ’18]

BACKGROUND AND MOTIVATION

- Orthogonal frequency division multiplexing (OFDM) has garnered widespread interests as the architectural backbone for communication-centric ISAC systems.

- **Features**
  - OFDM-based ISAC systems can achieve performance on par with sensing-centric systems, using dual-functional waveform for communications [Gaudio et al. ’19]
    - Sensing & communication signals occupy distinct frequency bands to mitigate interference, optimization typically revolves around power allocation across subcarriers
  - Multiple-input multiple-output (MIMO) involves beamforming techniques
    - MIMO Enables power allocations for beams directed towards both sensing receiver (SR) and communication receiver (CR)

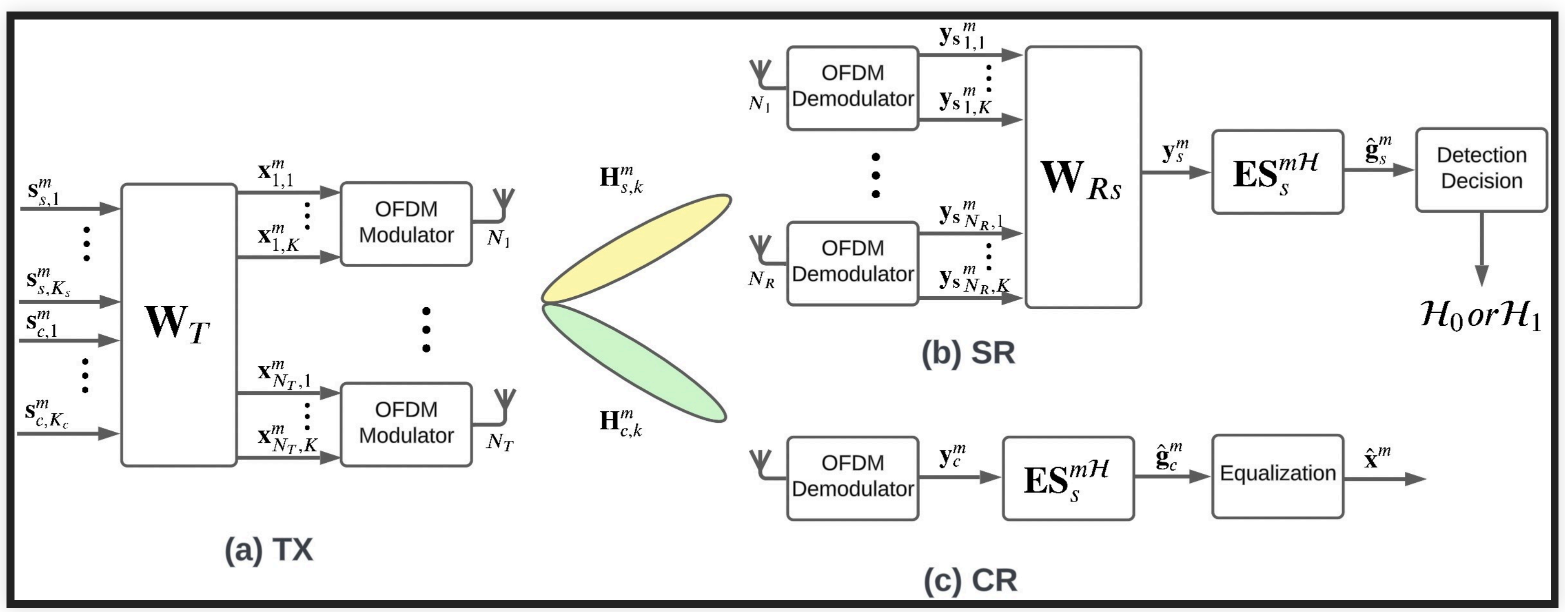
BACKGROUND AND MOTIVATION

- **Observation**
  - Characterization of PD and rate of MIMO-OFDM-based ISAC systems remains limited
  - Existing literature mainly focuses on Shannon capacity as performance metric of communication rate: It overlooks impact of imperfect channel information on detection performance and capacity
- **Question**
  - What are tradeoffs in ISAC incurred by joint power allocations across both multi-beam and subcarriers?
  - What is the joint performance bound when considering capacity with MMSE channel estimation?

OUTLINE

SYSTEM MODEL

- **System Model**
  - Non-collocated bi-static ISAC model based on MIMO-OFDM
  - $K \times 1$ OFDM block with superposition waveform model: $\mathbf{x}^m = \mathbf{s}_c^m + \mathbf{s}_s^m$ with power constraints $\|\mathbf{s}_s\|^2 = PP_s, \|\mathbf{s}_c\|^2 = PP_c$.
  - $N_T \times 1$ equivalent symbol vector on OFDM subcarrier k: [Pucci et al. '22]
    $\mathbf{x}_k^m = \mathbf{w}_T x_k^m = \left(\sqrt{\rho}\mathbf{w}_{Ts} + \sqrt{1-\rho}\mathbf{w}_{Tc}\right)x_k^m$
    $$\mathbf{w}_{Tc} = \frac{\sqrt{P_E}}{\|\boldsymbol{\alpha}^T(\theta_{Tc})\|_1}\boldsymbol{\alpha}^*(\theta_{Tc}), \mathbf{w}_{Ts} = \frac{\sqrt{P_E}}{\|\boldsymbol{\alpha}^T(\theta_{Ts})\|_1}\boldsymbol{\alpha}^*(\theta_{Ts}), \boldsymbol{\alpha}(\theta) = [\alpha_1(\theta), \alpha_2(\theta), \dots, \alpha_N(\theta)]^T.$$
  - **Intuition** : When sensing and communication signal occupy non-overlapping frequency resources, sensing signal serves as effective pilots for channel estimation.
  - Explicitly assume $|\mathcal{K}_s|$ subcarriers are allocated for sensing with dual purpose of channel estimation and detection



SYSTEM MODEL

- **Sensing Receiver**
  - Equivalent MIMO-OFDM system with beamforming.
  - $K \times 1$ frequency domain symbol vector at m-th OFDM block: $\mathbf{y}_s^m = \sqrt{\rho}\mathbf{G}_s^m\mathbf{x}^m + \mathbf{n}^m$
- **Communication Receiver**
  - Equivalent MISO-OFDM system with only transmit beamforming.
  - $K \times 1$ frequency domain symbol vector at m-th OFDM block: $\mathbf{y}_\mathrm{c}^m = \sqrt{1-\rho}\mathbf{G}_c\mathbf{x}^m + \boldsymbol{\eta}^m$
  - **Observation** : Both sensing and communication equivalent channels after OFDM demodulation are diagonal.
- **Removing Communication Interference**
  - Since sensing and communication signal occupy non-overlapping frequency resources, we can ensure $\mathbf{S}_s^{m\mathcal{H}}\mathbf{G}_s\mathbf{s}_c^m = \mathbf{0}$ by post-processing at the SR end, where $\mathbf{S}_s^m = \mathrm{diag}(\mathbf{s}_s^m)$, $\mathbf{g}_s = \mathrm{diag}(\mathbf{G}_s)$.
  - The interference from unknown communication signal can be removed as $\mathbf{S}_s^{m\mathcal{H}}\mathbf{y}_\mathrm{c}^m = \sqrt{1-\rho}\mathbf{S}_s^{m\mathcal{H}}\mathbf{S}_s^m\mathbf{g}_s + \mathbf{S}_s^{m\mathcal{H}}\boldsymbol{\eta}^m$.
  - At CR, the estimated channels across sensing subcarriers are instead used to interpolate communication subcarrier channels.

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PERFORMANCE METRICS

- **Communication performance metrics**
  - Most literature assumes perfect channel knowledge, where capacity of MISO-OFDM channel is achieved with power loading:

$$C = \frac{1}{K} \sum_{k \in \mathcal{K}_c} \mathcal{E}\left[\log\left(1 + \frac{\left|G_k^{(c)}\right|^2 (1-\rho) P_k^{(c)}}{\delta^2}\right)\right]$$

  - **Intuition** : Capacity with water-filling does not capture the situation with incomplete channel information
  - Capacity lower bound under MMSE channel estimation provides a more realistic tradeoff between rate and detection probability

Definition (Capacity under MMSE channel estimation)

$$C_{MMSE} \geq \frac{1}{K} \sum_{k \in \mathcal{K}_c} \mathcal{E}\left[\log\left(1 + \frac{\left|\hat{G}_k^{(c)}\right|^2}{\mathcal{E}\left[\left|G_k^{(c)} - \hat{G}_k^{(c)}\right|^2 + \frac{\delta^2}{(1-\rho)P_c}\right]}\right)\right]$$

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CLOSED FORM EXPRESSION FOR PD

Proposition (Probability of Detection)

The probability of detection has the following closed form:

$$P_D = Q_{K_s}\left(\sqrt{\frac{2\rho K_s(PP_s/K_s+\sigma_s^2)^2}{PP_s\sigma_s^2}\|\mathbf{E}\mathbf{S}_s^{m\mathcal{H}}\mathbf{S}_s^m\mathbf{g}_s^m\|^2}, \sqrt{\frac{2(\Gamma-\mu)K_s(PP_s/K_s+\sigma_s^2)^2}{\nu\sigma_s^2PP_s}}\right),$$

where

$$\mu = K_s\ln\left(\frac{2\sigma_s^2}{\rho PP_sP_E\sigma_h^2+\sigma_s^2}\right),$$

and

$$\nu = \frac{\left(PP_s/K_s+\sigma_s^2\right)^2}{\sigma_s^2PP_s} - \frac{\left(PP_s/K_s+\sigma_s^2\right)^2}{\sigma_s^2PP_s+\frac{\rho}{2}P^2P_s^2P_E\sigma_h^2}$$

CLOSED FORM EXPRESSION FOR PD

- **Proposition: Sketch of proof**
  - **Intuition** : Instead of performing hypothesis testing over received symbols, channel estimations are used as test statistics

$$\hat{\mathbf{g}}_s^m = \begin{cases} \mathcal{H}_1 : \sqrt{\rho}\mathbf{E}\mathbf{S}_s^{m\mathcal{H}}\mathbf{S}_s^m\mathbf{g}_s^m + \mathbf{E}\mathbf{S}_s^{m\mathcal{H}}\mathbf{n} \\ \mathcal{H}_0 : \mathbf{E}\mathbf{S}_s^{m\mathcal{H}}\mathbf{n} \end{cases}.$$

  - Formulate log-likelihood function as:

$$\ln\left[\frac{f\left(\hat{\mathbf{g}}_s^m|\mathcal{H}_1\right)}{f\left(\hat{\mathbf{g}}_s^m|\mathcal{H}_0\right)}\right] = \mu + \nu\|\hat{\mathbf{g}}_s^m\|^2$$

  - Noting that $\|\hat{\mathbf{g}}_s^m\|^2$ follows a non-central chi-squared distribution of degree $2K_s$ results in probability detection shown in Proposition and probability of false alarm (PFA) as

$$P_{FA} = Q_{K_s}\left(0, \sqrt{\frac{2(\Gamma-\mu)K_s(PP_s/K_s+\sigma_s^2)^2}{\nu\sigma_s^2 PP_s}}\right)$$

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NUMERICAL RESULTS

Unless otherwise noted, values for simulation parameters are summarized as follows.

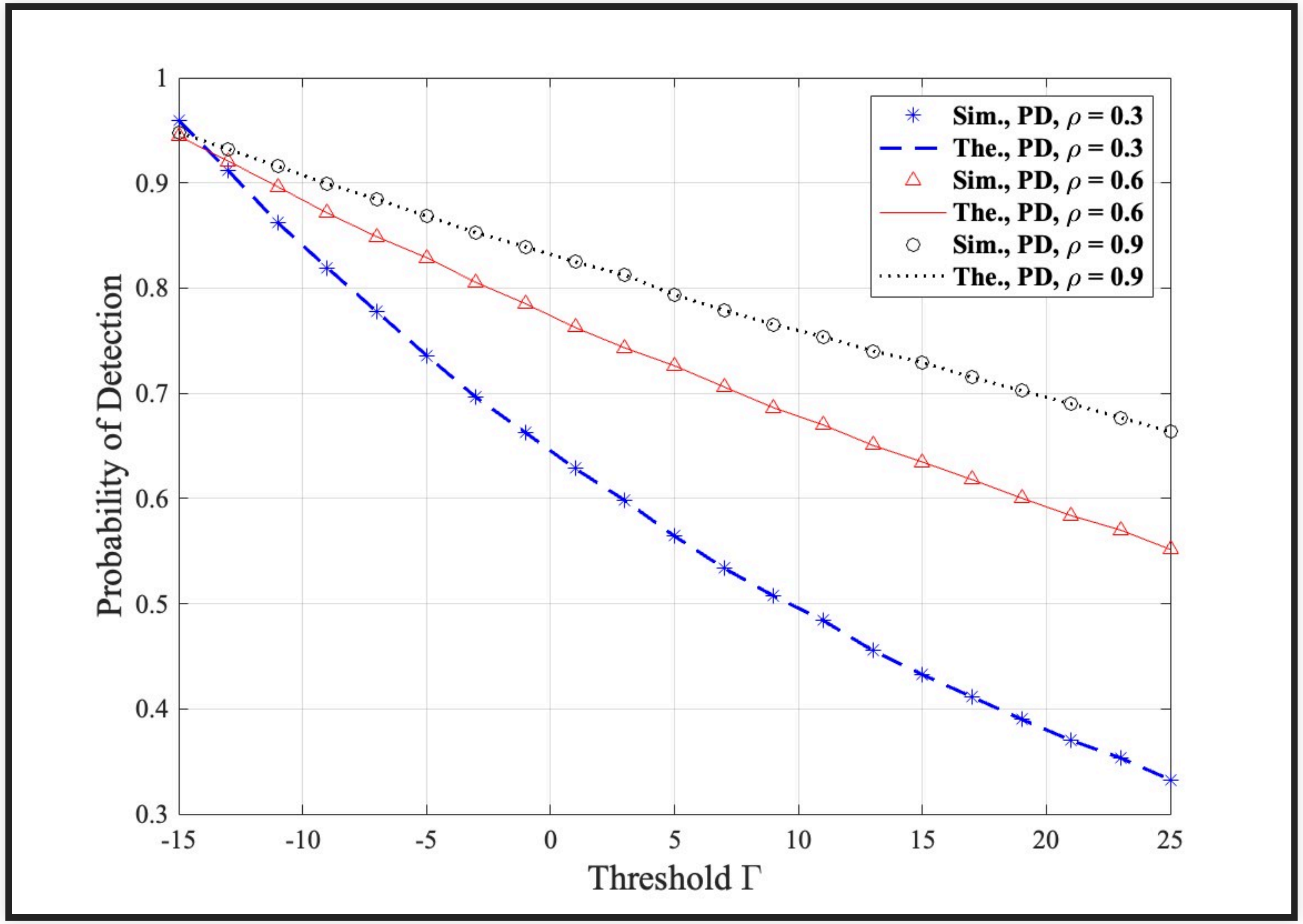
Parameter	**Value**
Carrier frequency f_c	3 GHz
Subcarrier frequency spacing $\triangle f$	0.59 MHz
Distance / pathloss from BS to CR/SR	100 m / 110 dB
Transmit power & EIPR	20 dBm
Modulation type	OFDM w/ QPSK
Total Subcarrier number K	64
Sensing Subcarrier number K_s	4
Noise power $\sigma_s^2=\sigma_s^2$	-100 dBm
Height of BS h_B	30 m
Height of receive antennas h_R	1 m
Channel realizations	100, 000 times

COST231 hata model pathloss:

$$L = (44.9 - 6.55\log_{10} h_B)\log_{10} d - (1.1\log_{10} f_c - 0.7)h_R + 5.83\log_{10} h_B + 35.46\log_{10} f_c - 89.2 \text{ dB}$$

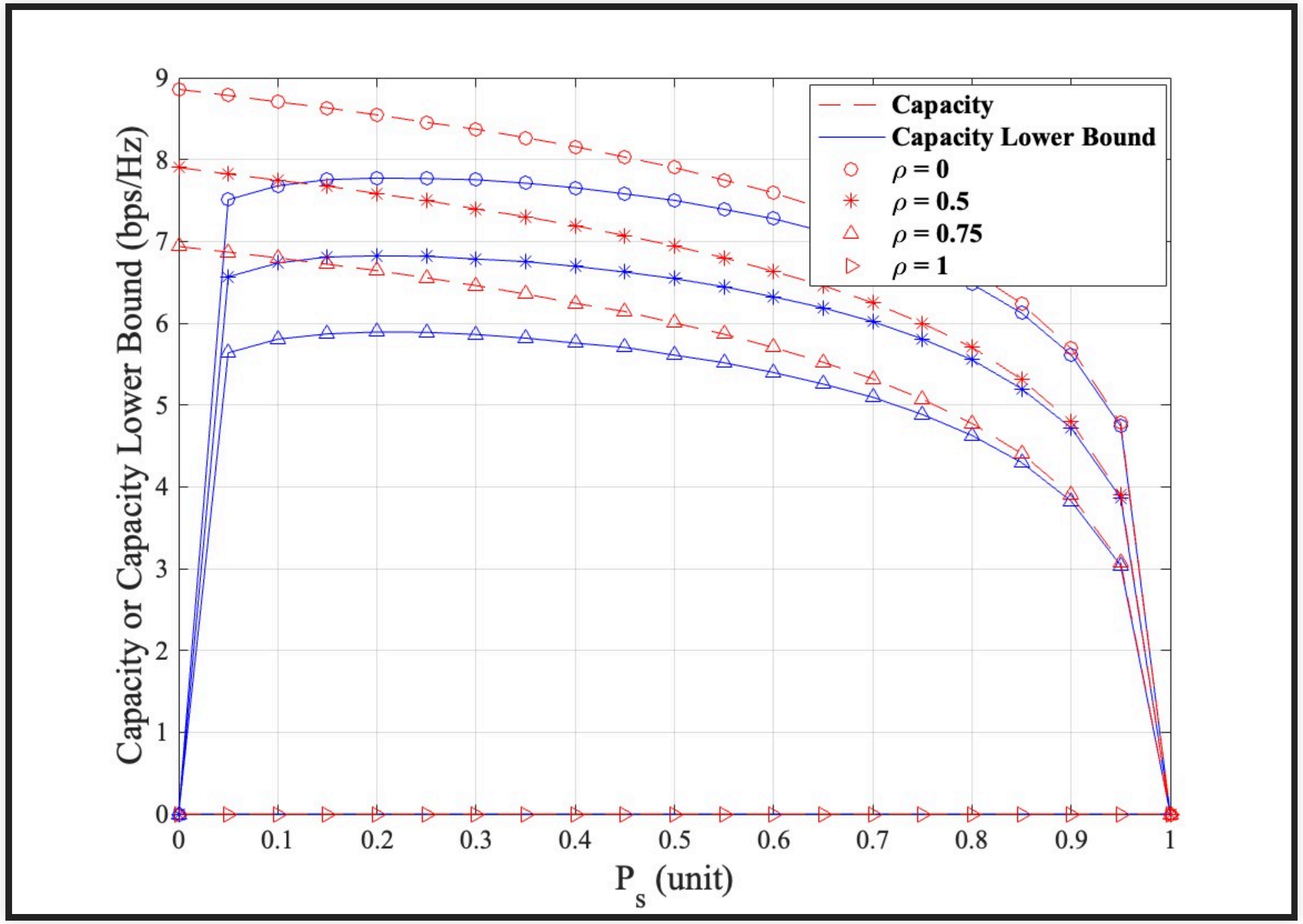
NUMERICAL RESULTS

- **Figure 2: Verification of accuracy of theoretical PD**
  - An equivalent receive SNR of 40 dB, with a subcarrier power splitting of $P_s = 0.5$.
  - PD decreases with increasing detection threshold
  - When more power is allocated for beamforming towards SR, increasing PD can be observed since sensing subsystem receives more power



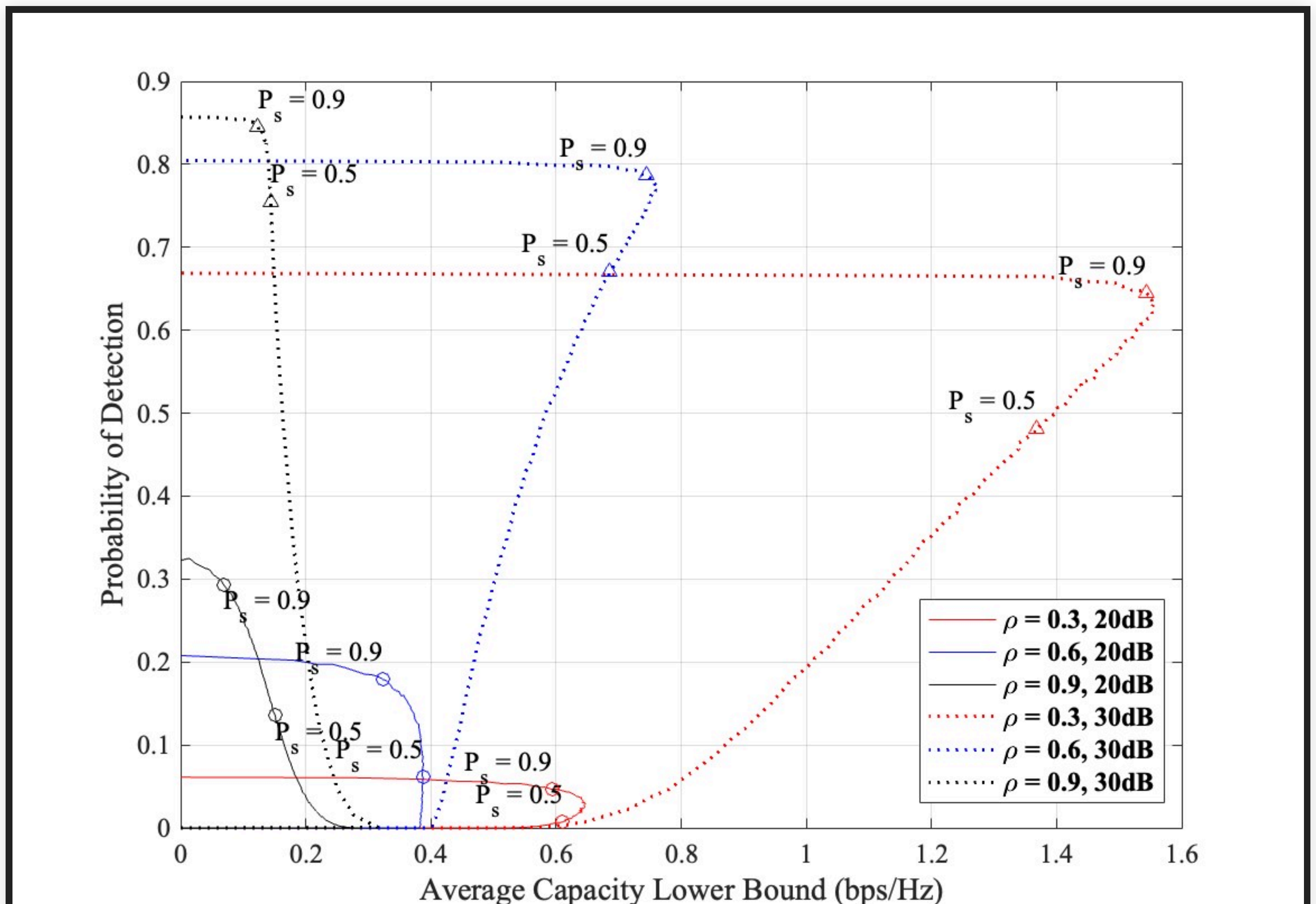
NUMERICAL RESULTS

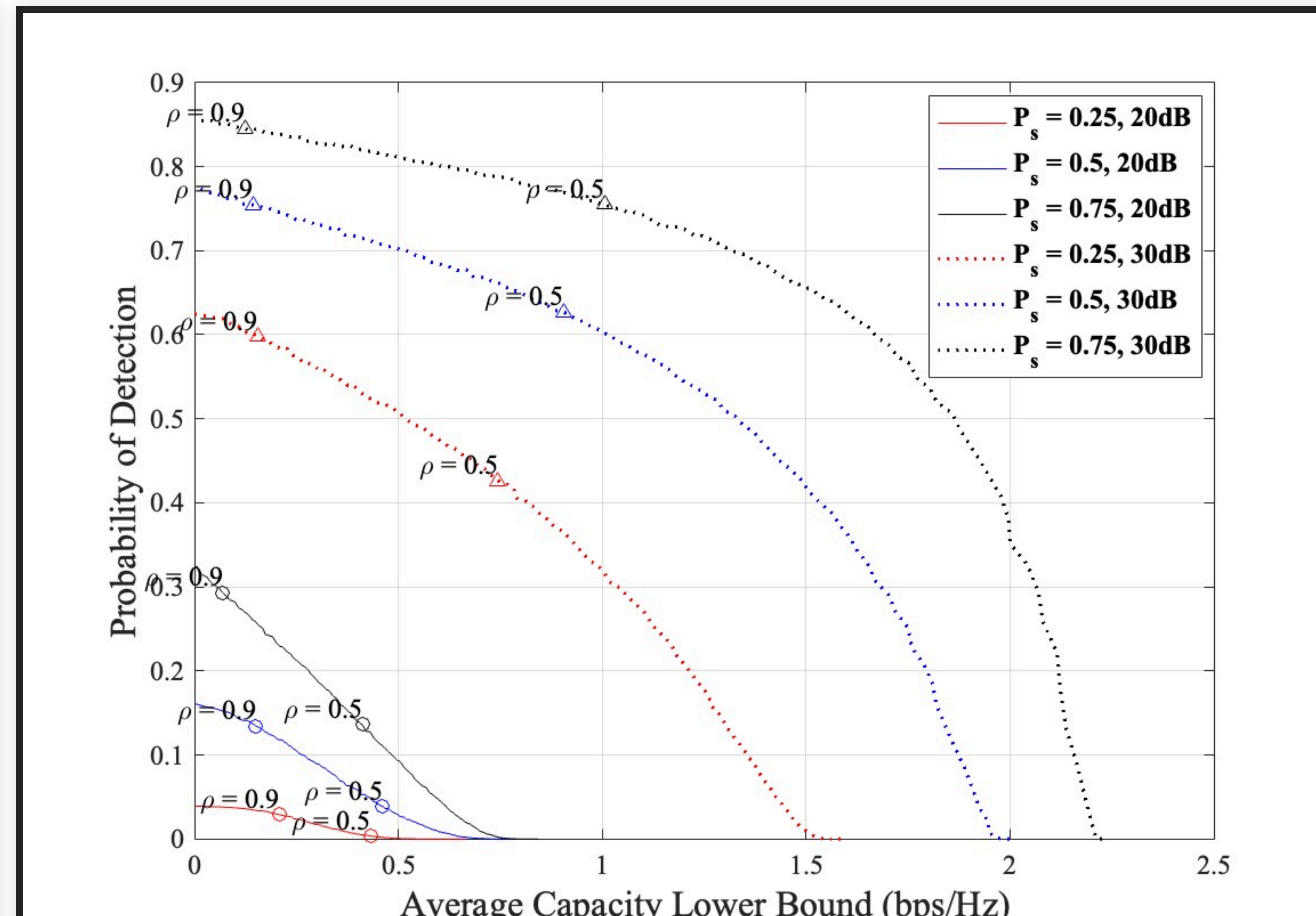
- **Figure 3: Relationship between capacity metrics & P_s**
  - An equivalent receive SNR of 40 dB, with a subcarrier power splitting of $P_s = 0.5$.
  - Both capacity metrics evidently see increases with decreasing ρ, as more power is used for beamforming towards CR
  - Inadequate power allocation towards sensing subcarriers → shannon capacity fails to account for uncertainty in channel estimation → disparity between two metrics at low P_s
  - Too much power allocated to sensing subcarriers, information-carrying subcarriers experience a decrease in SNR → reduced capacity



NUMERICAL RESULTS

- **Figure 4a,4b: Pareto regions for joint performance bound of capacity lower bound & PD**
  - PD approaches 1 with increasing total transmit power & fraction of power allocated for sensing beam ρ
  - Average capacity lower bound reaches a maximum at around $P_s = 0.9$; decreases as
    - More power for sensing subcarriers leads to insufficient resources for information-bearing symbols
    - Less power results in poor channel estimation
  - Different allocations of beam power $\rho \rightarrow$ non-overlapping performance bounds





CONCLUSION

- **We demonstrated performance tradeoff between PD and average capacity lower bound for an ISAC system based on MIMO-OFDM**
  - Optimization of joint performance bound is over fraction of power used for beamforming towards SR / CR, & fraction of power allocated for sensing / communication subcarriers.
- **We considered incomplete knowledge of channel information, and derived resulting exact PFA & PD expressions from hypothesis testing over channel estimations**
  - Joint performance bound → influence of joint resource allocation on communication & sensing functionalities.
- **Tradeoff observed**
  - Beamforming power allocation between rate & PD.
  - Subcarrier power allocation for pilot & message symbols → channel estimation quality, detection quality, and communication rate.

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Thank You

Email: sguo93@gatech.edu
xiangyuli@sjtu.edu.cn