RATE–RELIABILITY REGION OF SEQUENTIAL BEAM ALIGNMENT AND COMMUNICATION

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Sidong Guo and Matthieu R. Bloch

School of Electrical and Computer Engineering

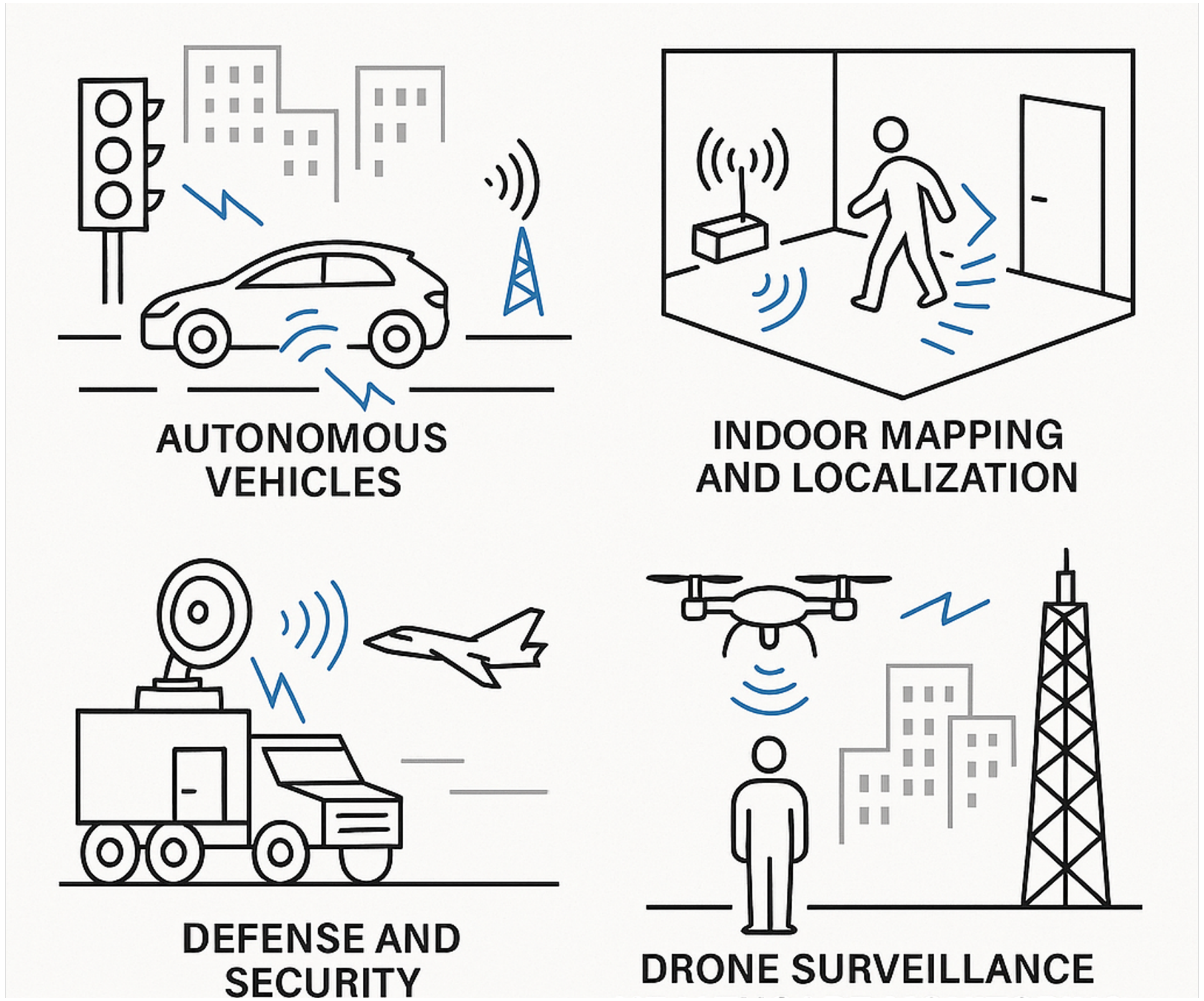
Georgia Institute of Technology

CONTEXT

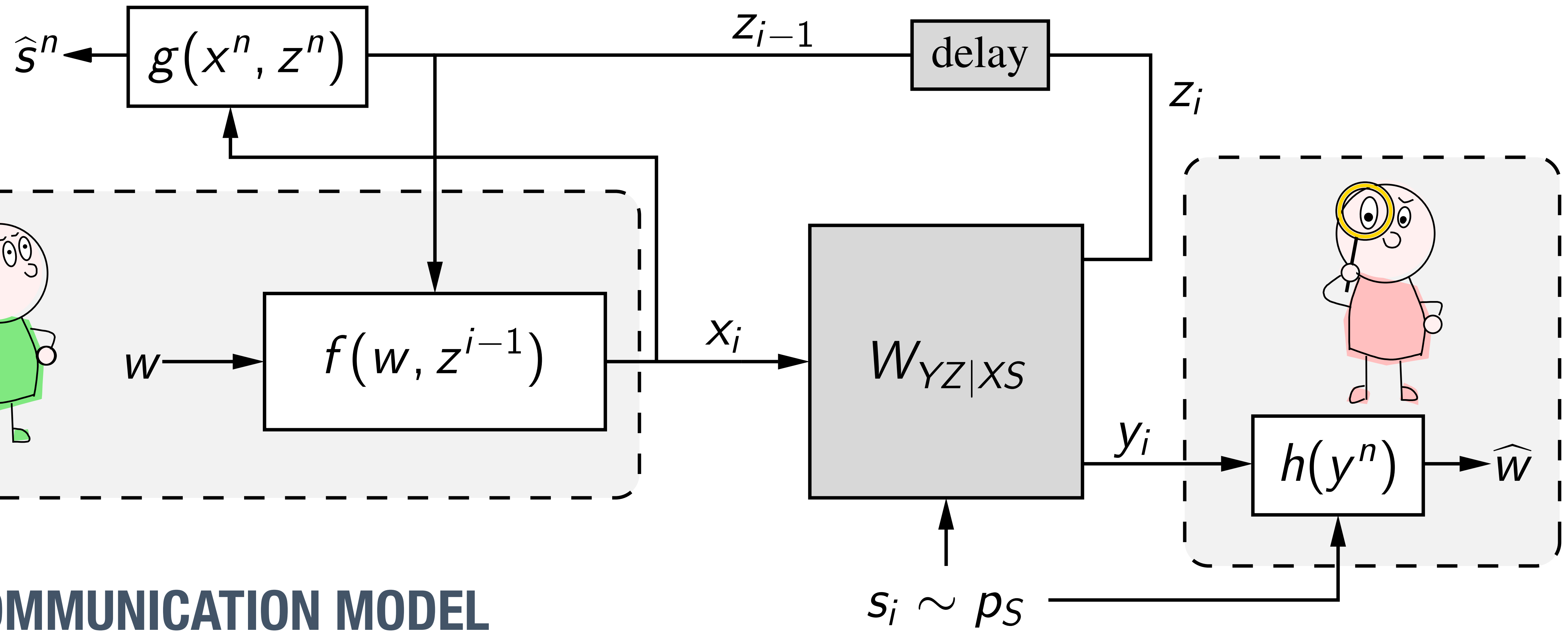
- Conventional approaches treat beam alignment and communication as separate phases, leading to significant overhead
- Existing joint beam alignment–communication models remain simplistic, especially in capturing the relative timing of wireless events and noise.

Summary

- We present a joint beam alignment and communication model using rateless coding with a space-time structure
- We analyze the communication rate and detection error exponent
- We present numerical results using a wireless example



▲ Illustration of JCAS use cases with dynamical states of interest (Image generated with the assistance of AI)



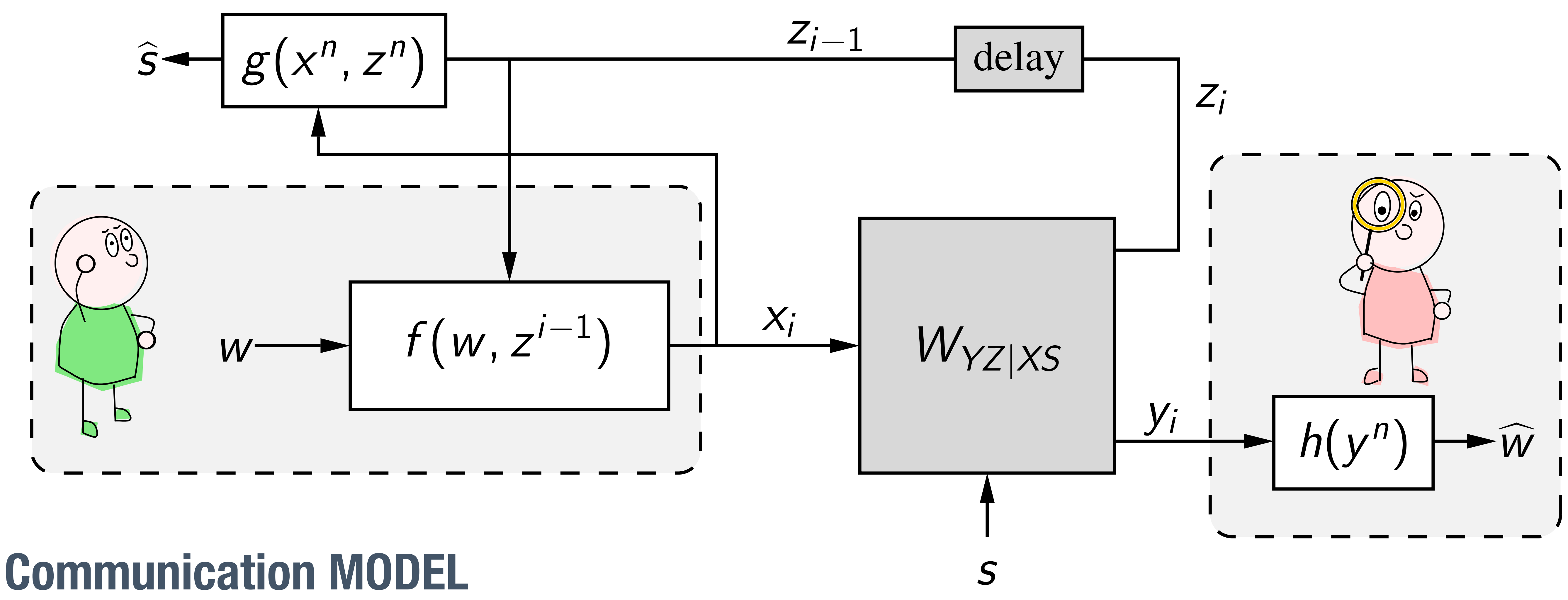
- **INTEGRATED SENSING AND COMMUNICATION MODEL**
  - **Metrics:**

$$P_e^{(n)} = \mathbb{P}\left(W \neq \widehat{W}\right) \quad d^{(n)} = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[d(S_i, \widehat{S}_i)\right]$$

  - **Reduces again to a rate-distortion tradeoff problem**
    - No prediction capability (state is i.i.d.)
    - Tradeoff governed by type of codewords $C(D) = \max_{p_X \in \mathcal{P}_D} I(X; Y|S)$

$$\mathcal{P}_D \triangleq \{\, p_X : \sum_x p_X(x) d^*(x) \leqslant D \,\} \quad d^*(x) \triangleq \mathbb{E}\left[d(S, \widehat{s}^*(X, Z)|X = x)\right] \quad \widehat{s}^*(x, z) \triangleq \operatorname*{argmin}_{s'} \sum_s p_{S|XZ} d(s, s')$$

Joint Transmission and State Estimation: A Constrained Channel Coding Approach, Zhang et al., *IEEE Transactions on Information Theory,* 2011
An Information-Theoretic Approach to Joint Sensing and Communication, Ahmadipour, Kobayashi, Wigger, Caire, *IEEE Trans. on Information Theory*, May 2022
An Information-Theoretic Approach to Collaborative Integrated Sensing and Communication for Two-Transmitter Systems, Ahmadipour and Wigger, *IEEE Journal on Selected Areas in Information Theory*, 2023



- **Integrated Sensing and Communication MODEL**
  - State-dependent Discrete Memoryless Channel (Compound Channel) with state $s \in \mathcal{S}$ $|\mathcal{S}| < \infty$
  - Encoder: $f_i : [1; M] \times \mathcal{Z}^{i-1} \to \mathcal{X}$ $\forall i \in [1; n]$, decoder: $h : \mathcal{Y}^n \to [1; M]$, estimator: $g : \mathcal{X}^n \times \mathcal{Z}^n \to \mathcal{S}$
- **Performance Metrics**
  - Communication and detection error probability:
    $$P_{\mathrm{c}}^{(n)} \triangleq \max_{s\in\mathcal{S}} \max_{w\in[1;M]} \mathbb{P}(h(Y^n) \neq w | W = w, S = s) \qquad P_{\mathrm{d}}^{(n)} \triangleq \max_{s\in\mathcal{S}} \frac{1}{M} \sum_{w=1}^{M} \mathbb{P}(g(Z^n) \neq s | S = s, W = w)$$
  - Rate: $R \triangleq \frac{1}{n} \log M$ and detection error exponent $E_{\mathrm{d}}^{(n)} \triangleq -\frac{1}{n} \log P_{\mathrm{d}}^{(n)}$

Rate and Detection Error-Exponent Tradeoffs of Joint Communication and Sensing, Chang et al., *Proc. of IEEE Int. Symp. on JC&S*, 2022
Joint Communication and Binary State Detection, Joudeh and Willems, *IEEE Journal on Selected Areas in Information Theory*, 2022
On Joint Communication and Channel Discrimination, Wu and Joudeh, *Proc. of IEEE International Symposium on Information Theory*, 2022
Rate and Detection-Error Exponent Tradeoff for Joint Communication and Sensing […], Chang et al., *IEEE Jour. on Sel. Areas in Info. Theory*, 2023
On the Fundamental Tradeoff of Integrated Sensing and Communications Under Gaussian Channels, Xiong et al., *IEEE Trans. on Info. Theory*, 2023

BEAMFORMING CODEBOOK IN MMWAVE INITIAL BEAM ALIGNMENT

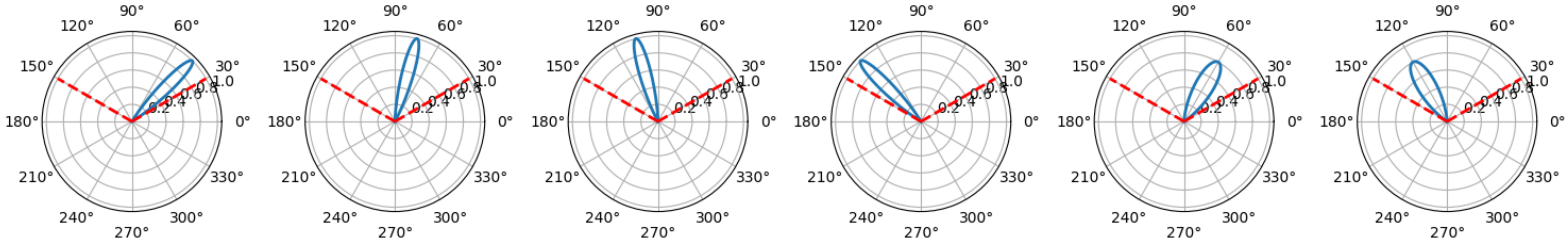
- Beamforming codewords at each level partition the angular search space with a given resolution δ **[1][2]**
- Larger beamwidth ⇒ lower signal-to-noise ratio (SNR)
- Beamforming vectors are sequentially selected to detect the user's direction

DETECTION PERFORMANCE OF HIERARCHICAL INITIAL BEAM ALIGNMENT

- Hierarchical beam alignment with fallback achieves the optimal detection error exponent **[3]**

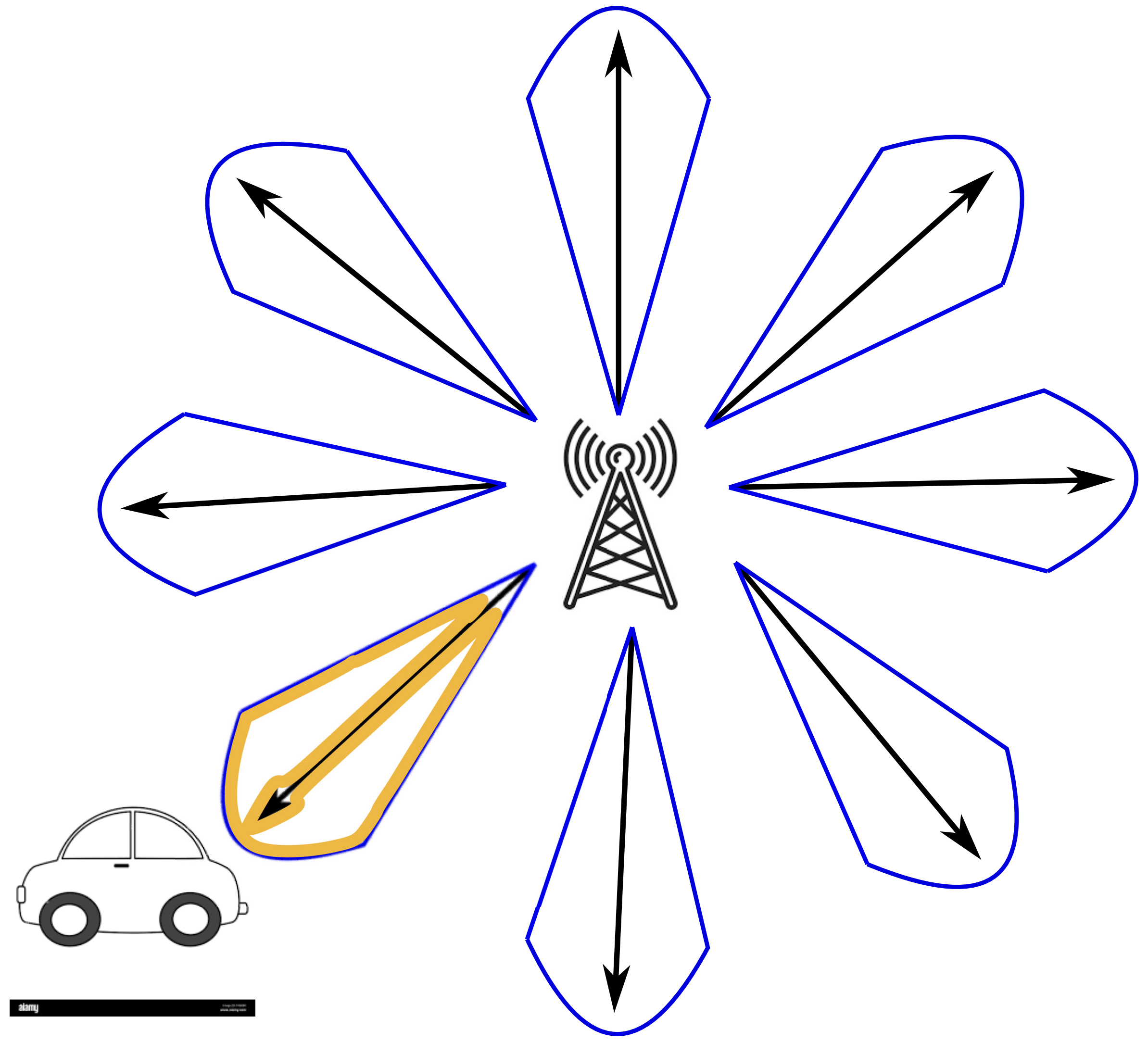
$$\mathbb{D}\left(\text{Bern}\left(p\left[\log_2\left(\frac{1}{\delta}\right)\right]\right)\middle\|\text{Bern}\left(1-p\left[\log_2\left(\frac{1}{\delta}\right)\right]\right)\right)$$

▼ Illustration of beam forming codebook



[1] Channel Estimation and Hybrid Precoding for Millimeter Wave Cellular Systems, A. Alkhateeb et al., *IEEE Trans. Signal Process. 2014*
[2] Multi-Armed Bandit Dynamic Beam zooming for mmWave Alignment and Tracking, N. Blinn and M. Bloch, *IEEE Trans. Wireless Commun.*, 2025
[3] Active Learning and CSI Acquisition for mmWave Initial Alignment, S. Chiu et al., *IEEE J. Sel. Areas Commun.*, 2019

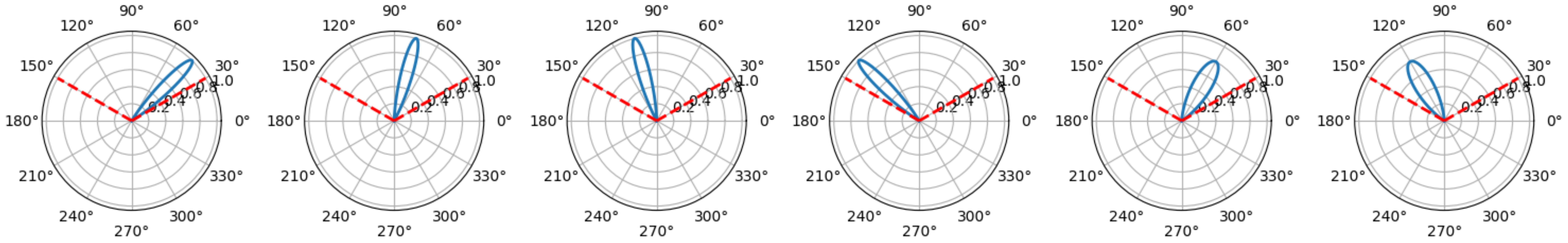
- **BINARY BEAM POINTING MODEL**
  - Noiseless toy model $Y = S \cdot X$
  - State is represented by an one-hot vector that “contains” the direction of target
  - Capacity is determined by the fraction of time true direction is probed
  - Peak of average cost constraint



▲ Illustration of beam pointing model [Li and Caire’23]

- **KEY INSIGHTS:**
  - Communication comes for free
  - Capacity is maximized using bisection-type target search
  - Optimal for direction detection

▼ Illustration of beam forming codebook

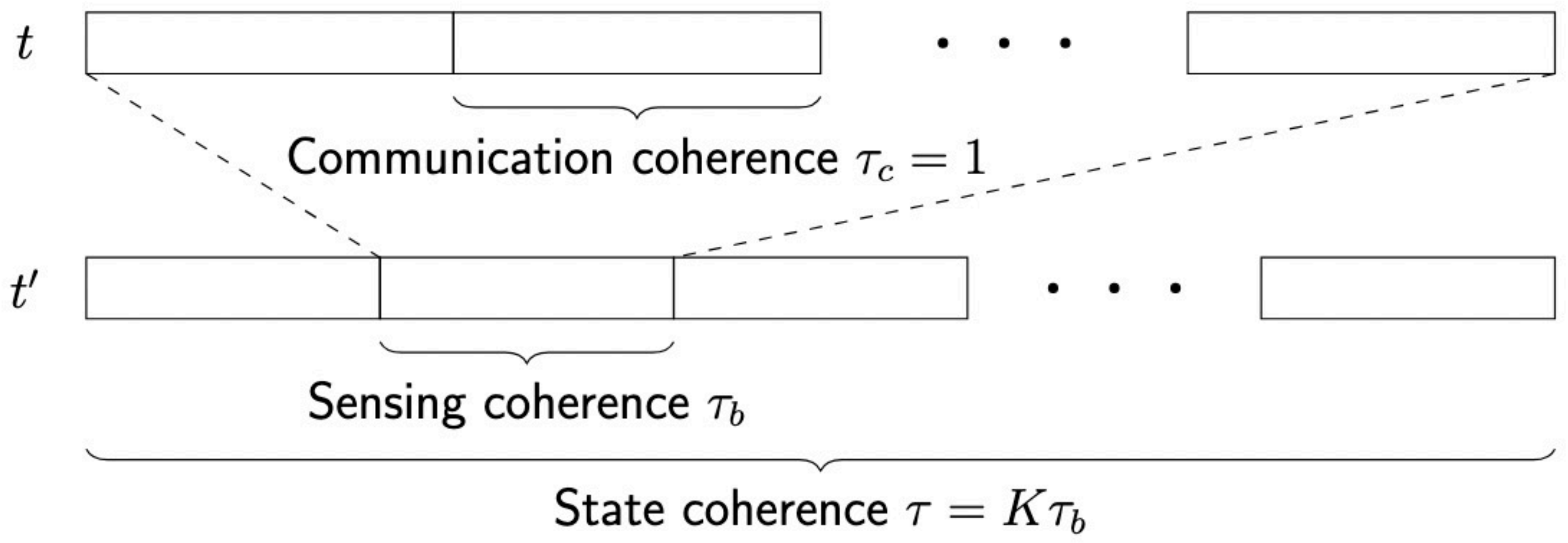


On the Capacity and State Estimation Error of “Beam-Pointing” Channel: The Binary Case, Li and Caire**,** *IEEE Trans. on Info. Theory*, 2023
On the Capacity of “Beam-Pointing” Channels with Block Memory and Feedback: The Binary Case , Li and Caire, *Proc. Asilomar Conf. Signals, Syst., Comput.*
On the Capacity of Gaussian “Beam-Pointing” Channels with Block Memory and Feedback, Siyao Li et al**,** *Proc. of 2024 ISIT*, 2024

- NOISY BEAM-POINTING CHANNEL MODEL: MODELING ASSUMPTIONS
  - **Finite-state hypothesis testing:** resolution $\delta = \Theta/B$
  - **One-hot state vector [Li-Caire'23]**: $S \in \mathcal{S} = \mathbb{F}_2^{1\times B}$
  - **Different sensing and communication timescale:** Assume Bernoulli noise model for $t' \in [1, \tau/\tau_b]$

$$Y_{\tau_b(t'-1)+1:\tau_b t'} = S \cdot X_{\tau_b(t'-1)+1:\tau_b t'} \oplus N_{\tau_b(t'-1)+1:\tau_b t'} \qquad X_{\tau_b t':\tau_b(t'+1)-1} \in \{0,1\}^{B\times\tau_b}$$

  - **One-bit sensing feedback [Chiu et al'19]:** $Z_{(t'+1)\tau_b} = \mathbf{1}\{\mathbf{1}^T Y^{\tau_b} > T\}$



On the Capacity and State Estimation Error of "Beam-Pointing" Channel: The Binary Case, Siyao Li and G. Caire, *IEEE Trans. on Info. Theory*, 2023
Active Learning and CSI Acquisition for mmWave Initial Alignment, S. Chiu et al., *IEEE J. Sel. Areas Commun.*, 2019
Sequential Joint Communication and Sensing of Fixed Channel States, M. Chang et al., *Proc. IEEE Inf. Theory Workshop (ITW), 2023*

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  - **One-bit sensing feedback [Chiu et al'19]:** $Z_{(t'+1)\tau_b} = \mathbf{1}\{\mathbf{1}^T Y^{\tau_b} > T\}$
  - **Space-time codeword matrix**

$$X^{\tau_b} = \underbrace{\begin{pmatrix} x_1(w) & x_2(w) & \dots & x_{\tau_b}(w) \\ x_1(w) & x_2(w) & \dots & x_{\tau_b}(w) \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{pmatrix}}_{\tau_b} \Big\updownarrow B \qquad a_{t'} = \{1,2\} \quad b = |a_{t'}| \qquad \alpha = \text{weight}(x_1(w), \cdots, x_{\tau_b}(w))$$

  - **Noise model:** $\mathbb{P}(Z_{(t'+1)\tau_b} \neq \mathbf{1}\{S \cdot X_{t'}^{\tau_b} \neq 0^{1\times\tau_b}\}) = p_b^{(\delta)}(\alpha) \qquad 0 < p_1^{(\delta)} < p_b^{(\delta)} < \cdots < p_B^{(\delta)} < 0.5$
    - Different input symbols correspond to varying signal strength

On the Capacity and State Estimation Error of "Beam-Pointing" Channel: The Binary Case, Siyao Li and G. Caire, *IEEE Trans. on Info. Theory*, 2023

Active Learning and CSI Acquisition for mmWave Initial Alignment, S. Chiu et al., *IEEE J. Sel. Areas Commun.*, 2019

Sequential Joint Communication and Sensing of Fixed Channel States, M. Chang et al., *Proc. IEEE Inf. Theory Workshop (ITW), 2023*

- **NOISY BEAM-POINTING CHANNEL MODEL: SEQUENTIAL VIEW**
  - $\phi_t : \mathbb{F}_2^{M_{t-1}} \times \mathcal{Z}^{t-1} \times \mathcal{X}^{t-1} \mapsto \mathbb{N}$ determines the number of transmitted messages
    - M_t non-decreasing and determines the number of messages transmitted by time
  - $f_t : \mathbb{F}_2^{M_t} \times \mathcal{Z}^{t-1} \times \mathcal{X}^{t-1} \mapsto \mathcal{X}$ determines the input to the channel
  - Stopping time criterion $\psi_t : \mathbb{F}_2^{M_t} \times \mathcal{Z}^t \times \mathcal{X}^t \mapsto \mathcal{B}$, where $\mathcal{B} = \{\text{continue}, \text{stop}\}$
  - State estimator $g : \mathbb{F}_2^{M_T} \times \mathcal{Z}^T \times \mathcal{X}^T \mapsto \Theta$
  - Message decoder $h : \mathcal{Y}^\tau \to \mathcal{M}$, where $\mathcal{M} = \bigcup_{k=1}^{\infty} \mathbb{F}_2^k$

- **PERFORMANCE METRICS**
  - Detection error $P_d^n \triangleq \max_{w \in \mathcal{M}} \mathbb{P}(\hat{S} \neq S \,|\, W = w)$
  - Communication error $P_c^n \triangleq \max_{w \in \mathcal{M}} \mathbb{P}(g(Y^\tau) \neq w[1; M_\tau] \,|\, W = w)$

- **RATIONALE**
  - Closed loop exponent for hypothesis testing unknown in fixed budget setting but know in sequential setting

Sequential Joint Communication and Sensing of Fixed Channel States, M. Chang et al., *Proc. IEEE Inf. Theory Workshop (ITW), 2023*

DEFINITION: ACHIEVABILITY

A policy is (R, E) achievable if for any $\epsilon_1, \epsilon_2, \epsilon_3 > 0$, there exists $n(\epsilon_1, \epsilon_2, \epsilon_3)$ such that

$$\max_{w\in\mathcal{M}} \mathbb{E}[\tau] \leq n, \min_{w\in\mathcal{M}} \mathbb{P}(\frac{M_\tau}{n} \geq R) \geq 1 - \epsilon_1,$$

$$P_c^n \leq \epsilon_2, -\frac{1}{n} \log P_d^n \geq E - \epsilon_3,$$

THEOREM: RATE AND EXPONENT REGION

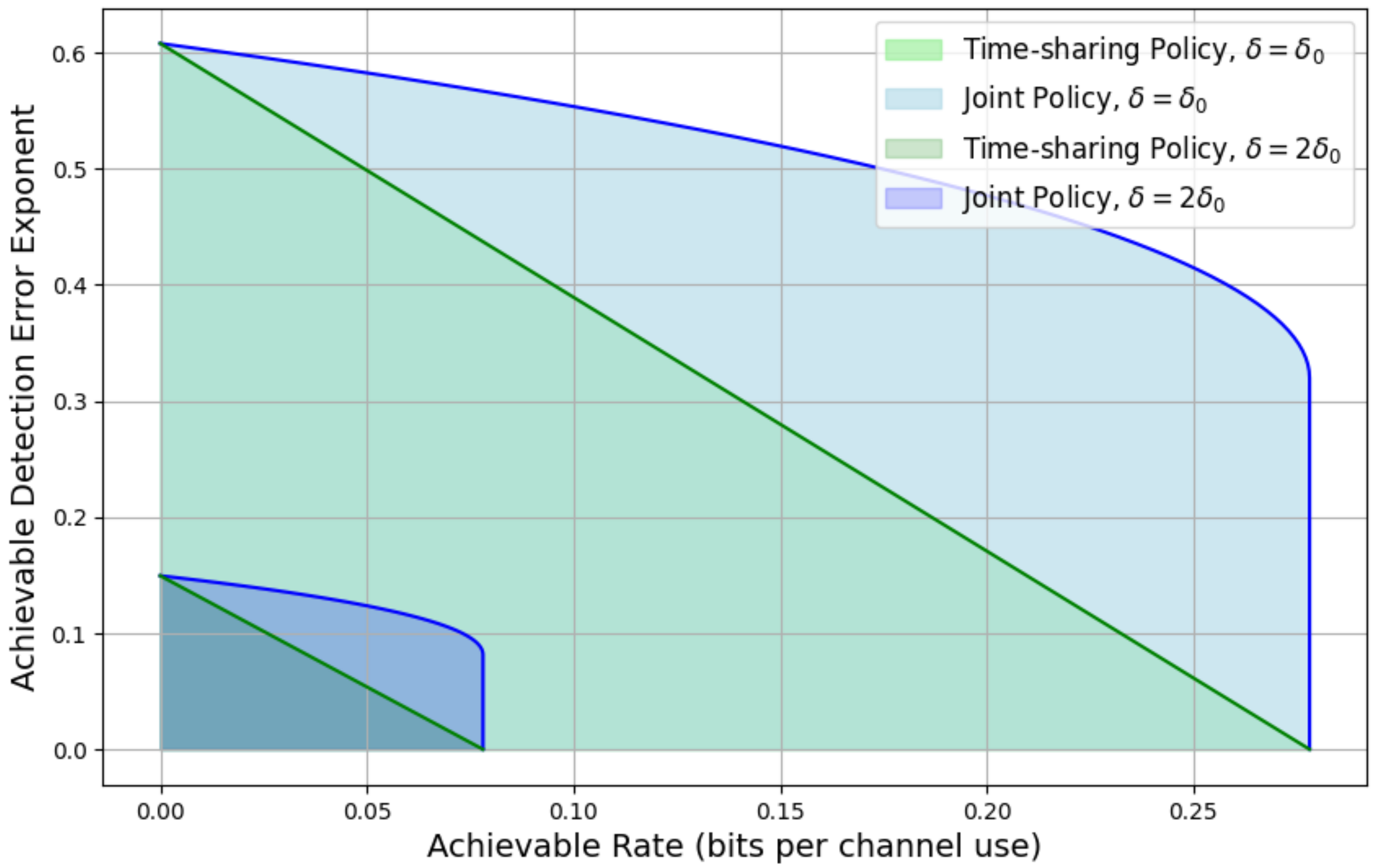
$$\mathcal{C} = \bigcup_{P_X\in\mathcal{P}_\mathcal{X}} \left\{ \begin{aligned} &(R, E) \in \mathbb{R}_+^2 : \\ &R \leq \mathbb{I}\left(P_X, W_{Y|X}(p_1^{(\delta)})\right) \\ &E \leq \frac{1}{\tau_b}\mathbb{D}\left(p_1^{(\delta)}(\alpha)||1 - p_1^{(\delta)}(\alpha)\right) \end{aligned} \right\}, \qquad p_{|a_{t'}|}^{(\delta)}(\alpha) = \mathbb{P}\left(Z_{(t'+1)\tau_b} \neq \mathbf{1}\left\{S \cdot X_{t'}^{\tau_b} \neq 0^{1\times\tau_b}\right\}\right)$$

- **KEY INSIGHTS:**
  - Joint beam alignment and communication incurs no loss in both detection and communication through adaptation
  - Still a tradeoff because of influence of α

- **SETUP**
  - Beam switching rate 200Hz, baud rate 10Gbps **[Paidimarri-Sadhu'20]**
  - $\tau_b = 50$
  - Base target resolution δ_0 $p_1^{(\delta_0)} = 0.2$, $p_1^{(2\delta_0)} \approx 0.337$,
    - based on $p_1^{(\delta)} \sim Q(1/\delta)$.
  - Gaussian approximation

$$p_1^{(\delta)}(\alpha) = 2Q\left(\frac{\alpha\tau_b(1-2p_1^{(\delta)})}{2\sqrt{\alpha\tau_b p_1^{(\delta)}(1-p_1^{(\delta)})}}\right)$$

- **OBSERVATIONS**
  - Finer resolution ⇒ better performance
  - outperforms time-sharing



Spatio-temporal filtering: Precise beam control using fast beam switching, Paidimarri and Sadhu, *IEEE Radio Frequency Integrated Circuits Symp., Jun, 2020.*

SKETCH OF PROOF

- **Policy**:
  - Spatial codeword: select ISAC action based on two phase policy **[1];**
  - Temporal codeword: constant composition code
- $$\eta^{(a)} = \begin{cases} \operatorname*{argmax}_{\lambda \in \mathbb{P}(\mathcal{A}_B)} \min_{\hat{\rho} \in \mathbb{P}(\Omega^B)} \sum_{a \in \mathcal{A}_B} \lambda_i^{(a)} \mathbb{D}(q_i^a \,||\, \sum_{j \neq i} \frac{\hat{\rho}_j}{1-\hat{\rho}_i} q_j^{a}) & \text{if } \exists i \ \rho_i \geq \rho*, \\ \operatorname*{argmax}_{\lambda \in \mathbb{P}(\mathcal{A}_B)} \min_{i \in \Omega^B} \min_{\hat{\rho} \in \mathbb{P}(\Omega^B)} \sum_{a \in \mathcal{A}_B} \lambda_i^{(a)} \mathbb{D}(q_i^a \,||\, \sum_{j \neq i} \frac{\hat{\rho}_j}{1-\hat{\rho}_i} q_j^{a}) & \text{if } \max_{i \in \Omega^B} \rho_i < \rho* . \end{cases}$$
- *Analysis of Stopping Time:* Formulate stopping rules based on achievable exponent to prove $\mathbb{E}[\tau] \leq n$
- *Analysis of Detection Exponent:* Follows the characterization in **[1]**
- *Analysis of Communication Error Probability:* union bound to express the error using three events
  - Decoding error in the first phase.
  - Decoding error in the second phase.
  - Wrong hypothesis crosses the threshold.
  - Each term shown to vanish

📚[1] **Active Sequential Hypothesis Testing,** Naghshvar and Javidi., *Annals of Statistics* (2013)
📚[2] **Extrinsic Jensen-ShannonDivergence: Applications to Variable-Length Coding,** Naghshvar et al.' *IEEE Transactions on Information Theory (2015)*

- SKETCH OF PROOF
  - **Analysis of Rate:**
    - Expected number of message bits transmitted by time t:
      - (Expected bits per block) × (Probability of that block in Exploration/Confirmation phase)
    - $M_\tau \geq \sum_{t'=0}^{\tau/\tau_b - 1} \mathbf{1}(\rho_{\tilde{i}}(t') \geq \rho^*)\lfloor \tau_b(\mathbb{I}(P_X^{\tau_b}, W_{Y|X}(p_1^{(\delta)})) - \epsilon)\rfloor + \mathbf{1}(\rho_{\tilde{i}}(t') < \rho^* \cap \tilde{i} \in a_{t'})\lfloor \tau_b(\mathbb{I}(P_X^{\tau_b}, W_{Y|X}(p_{|a|}^{(\delta)})) - \epsilon)\rfloor$
    - The time spent on first probing phase is small (bottom of the page).
    - The stopping time concentrates around n: $\mathbb{P}(\tau \geq (1-\zeta_1)n) \geq 1 - e^{-n\zeta_2}$
    - Probabilistic bound on rate: $\mathbb{P}(R^{(n)} \geq (1-\zeta_1)^2(\mathbb{I}(P_X^{\tau_b}, W_{Y|X}(p_1^{(\delta)})) - \epsilon)) \geq 1 - e^{-n\zeta_2}$.

Lemma: Martingale Concentration

Consider the sequence $U_{\tilde{i}}(t'), t' \in [0, \tau/\tau_b]$, there exists constants satisfying $0 < K_1 < K_2 < \infty$ such that

$$\mathbb{E}[U_{\tilde{i}}(t'+1) \,|\, \mathcal{F}_{t'}] \geq U_{\tilde{i}}(t') + K_1,$$
$$|U_{\tilde{i}}(t'+1) - U_{\tilde{i}}(t')| \leq K_2,$$

where $\mathcal{F}_{t'} = \sigma\{\boldsymbol{\rho}(t'), A(t'), Z(t'), t' \in [0, \tau/\tau_b]\}$.

Consider uniform prior and Markov stopping time, then $\mathbb{P}(U_{\tilde{i}}(n^{1/4}) < \theta) \leq e^{-n^{1/4}\iota}$ for $\iota, \theta \geq 0$.

- SKETCH OF PROOF
  - **Standard converse for the detection exponent [1]**
    - Applying a change of variable $t' = t/\tau_b$
    - Accounting for the relative timing of sensing and communication events
  - **Standard converse for rate**
    - Expected number of message bits transmitted by the stopping time upper-bounded by best channel
    - occurs when $|a| = 1$: $\mathbb{E}[M_\tau] \leq n(\mathbb{I}(P_X, W_{Y|X}(p^{(\delta)}_{|a|=1})) - \epsilon))$
    - best beamforming resolution (see also **[2]**).
    - Use Markov's inequality $\mathbb{P}(\frac{M_\tau}{n} \geq R) \geq 1 - \epsilon_1$ to convert expected number of transmitted message bits to a probabilistic constraint on rate $R \leq \mathbb{I}(P_X, W_{Y|X}(p_1^{(\delta)})) + o(\epsilon_1) - \epsilon$
    - Taking union of all action type P_X.

📚[1] **Active Sequential Hypothesis Testing,** Naghshvar and Javidi., *Annuls of Statistics* (2013)
📚[2] **Active Learning and CSI Acquisition for mmWave Initial Alignment,** S. Chiu et al., *IEEE J. Sel. Areas Commun.*, 2019

- **CONCLUSION**
  - Timing of wireless events important for modeling
  - Adaptation on long sensing coherence incurs no loss of sensing performance (in terms of exponent)
  - Transmitter can zoom in to achieve best exponent and rate as if direction were known **in hindsight**

- **LIMITATIONS & CONSIDERATIONS**
  - **Error propagation & latency**
    - Misalignment can lead to decoding errors, and retransmissions may increase latency
  - **Security vulnerability**
    - Joint beam alignment and communication may expose the system to new attack vectors

THANK YOU!